(1.2.) (1.3)

ПОГЛАВЉЕ 1.2 (N)

(1.) $z(x,y) = x^2 + xy - xy^3$

$z_x = 2x + y$

$z_y = x - 3y^2$

$$\boxed{dz = (2x+y)\,dx + (x-3y^2)\,dy}$$ ✓

(2.) $z(x,y) = \dfrac{1}{x^2+y^2}$

$z_x = \dfrac{-2x}{(x^2+y^2)^2}$

$z_y = \dfrac{-2y}{(x^2+y^2)^2}$

$$\boxed{dz = -\frac{2x}{(x^2+y^2)^2}\cdot dx - \frac{2y}{(x^2+y^2)^2}\cdot dy}$$ ✓

(3.) $z(x,y) = \dfrac{1}{x^2-y^2}$

$z_x = \dfrac{-2x}{(x^2-y^2)^2}$

$z_y = \dfrac{2y}{(x^2-y^2)^2}$

$$\boxed{dz = -\frac{2x}{(x^2-y^2)^2}\,dx + \frac{2y}{(x^2-y^2)^2}\,dy}$$ ✓

(4.) $z(x,y) = \dfrac{1}{x^2+y^2-x}$

$z_x = \dfrac{-(2x-1)}{(x^2+y^2-x)^2} = \dfrac{1-2x}{(x^2+y^2-x)^2}$

$z_y = \dfrac{-2y}{(x^2+y^2-x)^2}$

$$\boxed{dz = \frac{1-2x}{(x^2+y^2-x)^2}\,dx - \frac{2y}{(x^2+y^2-x)^2}\,dy}$$ ✓

(5.) $z(x,y) = xy + \sqrt{y}$

$z_x = y$

$z_y = x + \dfrac{1}{2\sqrt{y}}$

$$\boxed{dz = y\,dx + \left(x + \frac{1}{2\sqrt{y}}\right)dy}$$ ✓

⑥ $z(x,y) = \frac{x}{y} + \frac{y}{x}$

$z_x = \frac{1}{y} - \frac{y}{x^2}$

$z_y = -\frac{x}{y^2} + \frac{1}{x}$

$$\boxed{dz = \left(\frac{1}{y} - \frac{y}{x^2}\right)dx + \left(\frac{1}{x} - \frac{x}{y^2}\right)dy}$$ ✓

⑦ $z(x,y) = x^y$

$z_x = y x^{y-1}$

$z_y = x^y \ln x$

$$\boxed{dz = y x^{y-1}\,dx + x^y \ln x\,dy}$$ ✓

⑧ $z(x,y) = y^x, \quad M(1,1)$

$z_x = y^x \ln y = 0$

$z_y = x y^{x-1} = 1$

$$\boxed{dz = dy}$$ ✓

⑨ $z(x,y) = \sqrt{xy}; \quad M(2,2)$

$z_x = \frac{1}{2\sqrt{xy}} \cdot y = \frac{y}{2\sqrt{xy}} = \frac{1}{2}$

$z_y = \frac{1}{2\sqrt{xy}} \cdot x = \frac{x}{2\sqrt{xy}} = \frac{1}{2}$

$$\boxed{dz = \frac{1}{2}(dx + dy)}$$ ✓

⑩ $z(x,y) = \sqrt{\frac{x+1}{y-1}}; \quad M(3,2)$

$z_x = \frac{1}{2\sqrt{\frac{x+1}{y-1}}} \cdot \frac{1}{y-1} = \frac{\sqrt{y-1}}{2(y-1)\sqrt{x+1}} = \frac{1}{2\sqrt{(x+1)(y-1)}} = \frac{1}{4}$

$z_y = \frac{1}{2\sqrt{\frac{x+1}{y-1}}} \cdot \left(-\frac{x+1}{(y-1)^2}\right) \cdot 1 = -\frac{(y-1)}{2(x+1)}\frac{\sqrt{y-1}}{\sqrt{x+1}} = -\frac{(y-1)^{5/2}}{2(x+1)^{5/2}} = -\frac{1}{2^6} = -\frac{1}{64}$

Грешка

$$\boxed{dz = \frac{1}{4}dx - \frac{1}{64}dy}$$

$-1dy$

2

(11) $z(x,y) = xy\ln(xy)$, $M(1,e)$

$z_x = y\ln(xy) + \cancel{xy}\cdot\frac{1}{\cancel{xy}}\cdot y = y\ln(xy) + y = 2e$

$z_y = x\ln(xy) + \cancel{xy}\cdot\frac{1}{\cancel{xy}}\cdot x = x\ln(xy) + x = 2$

$\boxed{dz = 2e\,dx + 2dy}$ ✓

(12.) $z(x,y) = (x^3+y^4)^5$; $M(0,1)$

$z_x = 5(x^3+y^4)^4\cdot 3x^2 = 15x^2(x^3+y^4)^4 = 0$

$z_y = 5(x^3+y^4)^4\cdot 4y^3 = 20y^3(x^3+y^4)^4 = 20$

$\boxed{dz = 20dy}$ ✓

(13.) $z(x,y) = \operatorname{arctg}\frac{y}{x}$

$z_x = \frac{1}{1+\left(\frac{y}{x}\right)^2}\cdot\left(-\frac{y}{x^2}\right) = \frac{\cancel{x^2}}{x^2+y^2}\cdot\left(-\frac{y}{\cancel{x^2}}\right) = -\frac{y}{x^2+y^2}$

$z_y = \frac{1}{1+\left(\frac{y}{x}\right)^2}\cdot\frac{1}{x} = \frac{x^{\cancel{2}}}{x^2+y^2}\cdot\frac{1}{\cancel{x}} = \frac{x}{x^2+y^2}$

$\boxed{dz = -\frac{y}{x^2+y^2}dx + \frac{x}{x^2+y^2}dy}$ ✓

(14.) $z(x,y) = \operatorname{arctg}\frac{x}{y}$, $M(1,1)$

$z_x = \frac{y}{x^2+y^2} = \frac{1}{2}$

$z_y = -\frac{x}{x^2+y^2} = -\frac{1}{2}$

$\boxed{dz = \frac{1}{2}dx - \frac{1}{2}dy}$ ✓

(15.) $z(x,y) = x\operatorname{arctg}y$

$z_x = \operatorname{arctg}y$

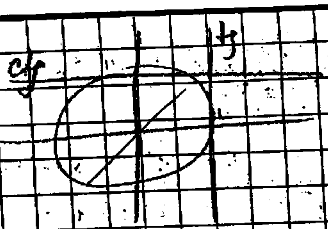
$z_y = x\cdot\frac{1}{1+y^2} = \frac{x}{1+y^2}$

$\boxed{dz = \operatorname{arctg}y\,dx + \frac{x}{1+y^2}dy}$ ✓

(16) $z(x,y) = y\operatorname{arctg}x$; $M(1,-1)$

$z_x = \frac{y}{1+x^2} = -\frac{1}{2}$

$z_y = \operatorname{arctg} x = \frac{\pi}{4} + k\pi$



$$\boxed{dz = -\frac{1}{2}dx + \left(\frac{\pi}{4}+k\pi\right)dy}$$

(17) $z(x,y) = \sqrt{x^2+y^2-4}$

$$z_x = \frac{1}{\cancel{2}\sqrt{x^2+y^2-4}} \cdot \cancel{2}x = \frac{x}{\sqrt{x^2+y^2-4}}$$

$$z_y = \frac{1}{\cancel{2}\sqrt{x^2+y^2-4}} \cdot \cancel{2}y = \frac{y}{\sqrt{x^2+y^2-4}}$$

$$\boxed{dz = \frac{x}{\sqrt{x^2+y^2-4}}\,dx + \frac{y}{\sqrt{x^2+y^2-4}}\,dy}$$

$z(x,y) = \ln(2x^2+y^2+y)$

(20) $z_x = \frac{1}{2x^2+y^2+y} \cdot 4x = \frac{4x}{2x^2+y^2+y}$

$$z_y = \frac{1}{2x^2+y^2+y} \cdot (2y+1) = \frac{2y+1}{2x^2+y^2+y}$$

$$\boxed{dz = \frac{4x}{2x^2+y^2+y}\,dx + \frac{2y+1}{2x^2+y^2+y}\,dy}$$

(19) $z(x,y) = \ln(9y-3x)$; $M(0,1)$

$$z_x = \frac{1}{9y-3x} \cdot (-3) = -\frac{3}{9y-3x} = -\frac{1}{3}$$

$$z_y = \frac{1}{9y-3x} \cdot 9 = \frac{9}{9y-3x} = 1$$

$$\boxed{dz = -\frac{1}{3}dx + dy}$$

(18) $z(x,y) = \sqrt{1-x^2} + \sqrt{1-y^2}$

$$z_x = \frac{1}{\cancel{2}\sqrt{1-x^2}} \cdot (-\cancel{2}x) = -\frac{x}{\sqrt{1-x^2}}$$

$$z_y = \frac{1}{\cancel{2}\sqrt{1-y^2}} \cdot (-\cancel{2}y) = -\frac{y}{\sqrt{1-y^2}}$$

$$\boxed{dz = -\frac{x}{\sqrt{1-x^2}}\,dx - \frac{y}{\sqrt{1-y^2}}\,dy}$$

(21.) $z(x,y) = \operatorname{arctg} \dfrac{x+y}{1+xy}$, $M(0,1)$

$$z_x = \frac{1}{1+\left(\frac{x+y}{1+xy}\right)^2} \cdot \frac{1+xy-(x+y)y}{(1+xy)^2} =$$

$$= \frac{\cancel{(1+xy)^2}}{(1+xy)^2+(x+y)^2} \cdot \frac{1+\cancel{xy}-\cancel{xy}-y^2}{\cancel{(1+xy)^2}} =$$

$$= \frac{1-y^2}{1+2xy+x^2y^2+x^2+2xy+y^2} =$$

$$= \frac{1-y^2}{1+4xy+x^2+y^2+x^2y^2} = 0$$

$$z_y = \frac{1}{1+\frac{(x+y)^2}{(1+xy)^2}} \cdot \frac{1+xy-(x+y)x}{(1+xy)^2} =$$

$$= \frac{\cancel{(1+xy)^2}}{(1+xy)^2+(x+y)^2} \cdot \frac{1+\cancel{xy}-x^2-\cancel{xy}}{\cancel{(1+xy)^2}} =$$

$$= \frac{1-x^2}{(1+xy)^2+(x+y)^2} = \frac{1}{2}$$

$$\boxed{dz = \frac{1}{2}dy}$$ ✓

(22.) $z(x,y) = \arcsin \dfrac{x+y}{1+xy}$

$$z_x = \frac{1}{\sqrt{1-\left(\frac{x+y}{1+xy}\right)^2}} \cdot \frac{1+xy-(x+y)y}{(1+xy)^2} =$$

$$= \frac{1}{\sqrt{\frac{(1+xy)^2-(x+y)^2}{(1+xy)^2}}} \cdot \frac{1+\cancel{xy}-\cancel{xy}-y^2}{(1+xy)^2} =$$

$$= \frac{\cancel{1+xy}}{\sqrt{(1+xy)^2-(x+y)^2}} \cdot \frac{1-y^2}{(1+xy)^{\cancel{2}}} =$$

$$= \frac{1-y^2}{(1+xy)\sqrt{(1+xy)^2-(x+y)^2}}$$

$$z_y = \frac{1}{\sqrt{1-\left(\frac{x+y}{1+xy}\right)^2}} \cdot \frac{1+xy-(x+y)x}{(1+xy)^2} =$$

$$= \frac{1+\cancel{xy}}{\sqrt{(1+xy)^2-(x+y)^2}} \cdot \frac{1+\cancel{xy}-x^2-\cancel{xy}}{(1+xy)^2} =$$

$$= \frac{1-x^2}{(1+xy)\sqrt{(1+xy)^2-(x+y)^2}}$$

$$\boxed{dz = \frac{1-y^2}{(1+xy)\sqrt{(1+xy)^2-(x+y)^2}}\,dx + \frac{1-x^2}{(1+xy)\sqrt{(1+xy)^2-(x+y)^2}}\,dy}$$

23) $z(x,y) = e^{\frac{x}{y}}$; $M(1,1)$

$z_x = e^{\frac{x}{y}} \cdot \frac{1}{y} = e$

$z_y = e^{\frac{x}{y}} \cdot \left(-\frac{x}{y^2}\right) = -e$

$$\boxed{dz = e\,dx - e\,dy}$$

24) $z(x,y) = y e^{\frac{x}{y}}$

$z_x = \cancel{y} e^{\frac{x}{y}} \cdot \frac{1}{\cancel{y}} = e^{\frac{x}{y}}$

$z_y = e^{\frac{x}{y}} + \cancel{y} e^{\frac{x}{y}} \cdot \left(-\frac{x}{y^{\cancel{2}}}\right) = e^{\frac{x}{y}} - \frac{x}{y} e^{\frac{x}{y}} = e^{\frac{x}{y}}\left(1 - \frac{x}{y}\right)$

$$\boxed{dz = e^{\frac{x}{y}} \cdot dx + e^{\frac{x}{y}}\left(1 - \frac{x}{y}\right) dy}$$

25. [redacted] $\rightarrow$ sh, ch?

29) $z(x,y) = \arcsin \frac{y}{x}$; $M(1,1)$

$z_x = \frac{1}{\sqrt{1-\left(\frac{y}{x}\right)^2}} \cdot \left(-\frac{y}{x^2}\right) = \frac{x}{\sqrt{x^2-y^2}} \cdot \left(-\frac{y}{x^2}\right) = -\frac{y}{x\sqrt{x^2-y^2}} = -\infty$

$z_y = \frac{1}{\sqrt{1-\left(\frac{y}{x}\right)^2}} \cdot \frac{1}{x} = \frac{x}{\sqrt{x^2-y^2}} \cdot \frac{1}{x} = \frac{1}{\sqrt{x^2-y^2}} = +\infty$

$dz = ?$

30) $z(x,y) = \arccos \frac{x}{x+y}$

$z_x = -\frac{1}{\sqrt{1-\left(\frac{x}{x+y}\right)^2}} \cdot \frac{\cancel{x}+y-\cancel{x}}{(x+y)^2} = -\frac{\cancel{x+y}}{\sqrt{(x+y)^2-x^2}} \cdot \frac{y}{(x+y)^{\cancel{2}}}$

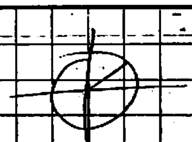
$= -\frac{y}{(x+y)\sqrt{(x+y+x)(\cancel{x+y-x})}} = -\frac{y}{(x+y)\sqrt{y(2x+y)}}$

$z_y = -\frac{1}{\sqrt{1-\left(\frac{x}{x+y}\right)^2}} \cdot \left(-\frac{x}{(x+y)^2}\right) = -\frac{\cancel{x+y}}{\sqrt{(x+y)^2-x^2}} \cdot \left(-\frac{x}{(x+y)^{\cancel{2}}}\right)$

$= \frac{x}{(x+y)\sqrt{y(2x+y)}}$

$$\boxed{dz = -\frac{y}{(x+y)\sqrt{y(2x+y)}}\,dx + \frac{x}{(x+y)\sqrt{y(2x+y)}}\,dy}$$

(31.) $z(x,y) = \frac{1}{\sin x} - \frac{1}{\cos y}$; $M(\pi/4, \pi/4)$



$z_x = -\frac{1}{\sin^2 x} \cdot \cos x = -\frac{\cos x}{\sin^2 x} = -\frac{\frac{\sqrt{2}}{2}}{\frac{2}{4}} = -\frac{4\sqrt{2}}{4} = -\sqrt{2}$

$z_y = \frac{1}{\cos^2 y}(-\sin y) = -\frac{\sin y}{\cos^2 y} = -\sqrt{2}$

$\boxed{dz = -\sqrt{2}\,dx - \sqrt{2}\,dy}$

(32.) $z(x,y) = \ln(y - \sin(xy+y))$

$z_x = \frac{1}{y - \sin(xy+y)} \cdot (-\cos(xy+y)) \cdot y = -\frac{y \cdot \cos(xy+y)}{y - \sin(xy+y)}$

$z_y = \frac{1}{y - \sin(xy+y)} \cdot (1 - \cos(xy+y)) \cdot (x+1) = \frac{1-(x+1)(\cos(xy+y))}{y - \sin(xy+y)}$

$\boxed{dz = -\frac{y\cos(xy+y)}{y - \sin(xy+y)}\,dx + \frac{1-(x+1)\cos(xy+y)}{y - \sin(xy+y)}\,dy}$

(33.) $z(x,y) = \sqrt{2x - x^2 - y^2}$

$z_x = \frac{1}{2\sqrt{2x-x^2-y^2}} \cdot (2-2x) = \frac{2(1-x)}{2\sqrt{2x-x^2-y^2}} = \frac{1-x}{\sqrt{2x-x^2-y^2}}$

$z_y = \frac{1}{2\sqrt{2x-x^2-y^2}} \cdot (-2y) = -\frac{y}{\sqrt{2x-x^2-y^2}}$

$\boxed{dz = \frac{1-x}{\sqrt{2x-x^2-y^2}}\,dx - \frac{y}{\sqrt{2x-x^2-y^2}}\,dy}$

(34.) $z(x,y) = (\sin x)^{\cos y}$

$z_x = \cos y \cdot \sin x^{(\cos y - 1)} \cdot \cos x$

$z_y = (\sin x)^{\cos y} \cdot \ln \sin x$

$\boxed{dz = \cos y\,(\sin x)^{(\cos y - 1)} \cdot \cos x\,dx + (\sin x)^{\cos y} \cdot \ln(\sin x)\,dy}$

(35.) $z(x,y) = (tg\,y)^{\cos x}$

$z_x = (tg\,y)^{\cos x} \cdot \ln tg\,y \cdot (-\sin x)$

$z_y = \cos x \cdot (tg\,y)^{(\cos x - 1)} \cdot \frac{1}{\cos^2 y} = \frac{\cos x}{\cos^2 y} \cdot (tg\,y)^{(\cos x - 1)}$

$\boxed{dz = (tg\,y)^{\cos x} \cdot \ln(tg\,y) \cdot (-\sin x)\,dx + \frac{\cos x}{\cos^2 y}\,(tg\,y)^{(\cos x - 1)}\,dy}$

36. $z(x,y) = y\sin x + x\cos y$

$z_x = y\cos x + \cos y$

$z_y = \sin x - x\sin y$

$$\boxed{dz = (y\cos x + \cos y)dx + (\sin x - x\sin y)dy}$$

37. $z(x,y) = \frac{1}{\operatorname{sh} x} - \frac{y}{\operatorname{ch} y}$

$z_x = -\frac{1}{\operatorname{sh}^2 x}\cdot \operatorname{ch} = -\frac{\operatorname{ch}}{\operatorname{sh}^2 x}$

$z_y = \frac{-\operatorname{ch} y + y\operatorname{sh} y}{\operatorname{ch}^2 y} = \frac{y\operatorname{sh} y - \operatorname{ch} y}{\operatorname{ch}^2 y}$

$$\boxed{dz = -\frac{\operatorname{ch}}{\operatorname{sh}^2 x}dx + \frac{y\operatorname{sh} y - \operatorname{ch} y}{\operatorname{ch}^2 y}dy}$$

38. $z(x,y) = \frac{x - y^2}{x^2 + y^3}$

$z_x = \frac{x^2 + y^3 - (x - y^2)\cdot 2x}{(x^2+y^3)^2} = \frac{x^2+y^3-2x^2+2xy^2}{(x^2+y^3)^2} = \frac{-x^2+y^3+2xy^2}{(x^2+y^3)^2}$

$z_y = \frac{-2y(x^2+y^3) - (x-y^2)\cdot 3y^2}{(x^2+y^3)^2} = \frac{-2yx^2-2y^4-3xy^2+3y^4}{(x^2+y^3)^2} =$

$= \frac{y^4-2yx^2-3xy^2}{(x^2+y^3)^2}$

$$\boxed{dz = \frac{-x^2+y^3+2xy^2}{(x^2+y^3)^2}dx + \frac{y^4-2yx^2-3xy^2}{(x^2+y^3)^2}dy}$$

39. $z(x,y) = \frac{1}{\operatorname{tg} x} + \frac{xy}{\sin y}$

$z_x = -\frac{1}{\operatorname{tg}^2 x}\cdot\frac{1}{\cos^2 x} + \frac{y}{\sin y} = -\frac{\cos^2 x}{\sin^2 x}\cdot\frac{1}{\cos^2 x} + \frac{y}{\sin y} = -\frac{1}{\sin^2 x} + \frac{y}{\sin y}$

$z_y = \frac{x\sin y - xy\cos y}{\sin^2 y} = \frac{x(\sin y - y\cos y)}{\sin^2 y}$

$$\boxed{dz = \left(\frac{y}{\sin y} - \frac{1}{\sin^2 x}\right)dx + \frac{x(\sin y - y\cos y)}{\sin^2 y}dy}$$

(40) $z(x,y) = \dfrac{x+\sqrt{y}}{\sqrt{x}+y^3}$, $M(1,1)$

$$Z_x = \frac{\sqrt{x}+y^3 - (x+\sqrt{y})\cdot\frac{1}{2\sqrt{x}}}{(\sqrt{x}+y^3)^2} = \frac{(\sqrt{x}+y^3)\,2\sqrt{x} - x - \sqrt{y}}{2\sqrt{x}(\sqrt{x}+y^3)^2}$$

$$= \frac{2x + 2\sqrt{x}\cdot y^3 - x - \sqrt{y}}{2\sqrt{x}(\sqrt{x}+y^3)^2} = \frac{x - \sqrt{y} + 2\sqrt{x}\cdot y^3}{2\sqrt{x}(\sqrt{x}+y^3)^2} = \frac{1}{4}$$

$$Z_y = \frac{\frac{1}{2\sqrt{y}}(\sqrt{x}+y^3) - (x+\sqrt{y})\cdot 3y^2}{(\sqrt{x}+y^3)^2} = \frac{\sqrt{x}+y^3 - 6\sqrt{y}\cdot y^2(x+\sqrt{y})}{2\sqrt{y}(\sqrt{x}+y^3)^2} = -\frac{5}{4}$$

$$\boxed{dz = \frac{1}{4}dx - \frac{5}{4}dy}$$ ✓

(41) $z(x,y) = \frac{1}{x}\arcsin(xy)$

$$Z_x = -\frac{1}{x^2}\arcsin(xy) + \frac{1}{x}\cdot\frac{1}{\sqrt{1-(xy)^2}}\cdot y =$$

$$= -\frac{1}{x^2}\arcsin(xy) + \frac{y}{x}\cdot\frac{1}{\sqrt{1-(xy)^2}}$$

$$Z_y = \frac{1}{\cancel{x}}\cdot\frac{1}{\sqrt{1-(xy)^2}}\cdot\cancel{x} = \frac{1}{\sqrt{1-(xy)^2}}$$

$$\boxed{dz = \frac{1}{x}\left(\frac{y}{\sqrt{1-(xy)^2}} - \frac{\arcsin(xy)}{x}\right)dx + \frac{1}{\sqrt{1-(xy)^2}}\,dy}$$ ✓

(42) $z(x,y) = y\cdot\arccos\frac{x}{y}$; $M(0,1)$

$$Z_x = \cancel{y}\cdot\left(-\frac{1}{\sqrt{1-\left(\frac{x}{y}\right)^2}}\right)\cdot\frac{1}{\cancel{y}} = -\frac{y}{\sqrt{1-x^2}} = -1$$

$y^2 - x^2$

$$Z_y = \arccos\frac{x}{y} + \cancel{y}\cdot\left(-\frac{1}{\sqrt{1-\left(\frac{x}{y}\right)^2}}\right)\cdot\left(-\frac{x}{y^2}\right) =$$

$$= \arccos\frac{x}{y} + \frac{x\,y}{y\sqrt{1-x^2}} = \arccos\frac{x}{y} + \frac{x}{\sqrt{1-x^2}} = \frac{\pi}{2} + k\pi$$

$y^2 - x^2$

$$\boxed{dz = -dx + \left(\frac{\pi}{2} + k\pi\right)dy}$$ ✓

(43) $z(x,y) = \arcsin\frac{x}{y^2} + \arcsin(1-y)$

$$Z_x = \frac{1}{\sqrt{1-\left(\frac{x}{y^2}\right)^2}}\cdot\frac{1}{\cancel{y^2}} = \frac{\cancel{y^2}}{\sqrt{y^4 - x^2}}\cdot\frac{1}{\cancel{y^2}} = \frac{1}{\sqrt{y^4 - x^2}}$$

$$Z_y = \frac{1}{\sqrt{1-\left(\frac{x}{y^2}\right)^2}}\cdot\left(-\frac{2x}{y^3}\right) + \frac{1}{\sqrt{1-(1-y)^2}}\cdot(-1) =$$ ✓

$$= \frac{y^2}{\sqrt{1-x^2}}\cdot\left(-\frac{2x}{y^3}\right) - \frac{1}{\sqrt{(1+1-y)(1-1-y)}} -$$

$$= -\frac{2x}{\sqrt{1-x^2}} - \frac{1}{\sqrt{y(y-2)}}$$

$$\boxed{dz = \frac{1}{\sqrt{y^2-x^2}}\,dx - \left(\frac{2x}{\sqrt{y^2-x^2}} + \frac{1}{\sqrt{y(y-2)}}\right)dy}$$

44. $z(x,y) = x\sin^2\frac{x}{y}$

$z_x = \sin^2\frac{x}{y} + x\cdot 2\sin\frac{x}{y}\cdot\cos\frac{x}{y}\cdot\frac{1}{y} = \sin^2\frac{x}{y} + \frac{x}{y}\sin\frac{2x}{y}$

$z_y = x\cdot 2\sin\frac{x}{y}\cdot\cos\frac{x}{y}\left(-\frac{x}{y^2}\right) = -\frac{x^2}{y^2}\sin\frac{2x}{y}$

$$\boxed{dz = \left(\sin^2\frac{x}{y} + \frac{x}{y}\cdot\sin\frac{2x}{y}\right)dx - \frac{x^2}{y^2}\sin\frac{2x}{y}\,dy}$$

$2\sin\alpha\cos\alpha = \sin 2\alpha$

[redacted] $u(x,y,z) = \ln(x^2+y^2-z)$

47. $u_x = \frac{1}{x^2+y^2+z}\cdot 2x = \frac{2x}{x^2+y^2-z}$

$u_y = \frac{2y}{x^2+y^2-z}$

$u_z = -\frac{1}{x^2+y^2-z}$

$$\boxed{du = \frac{2x}{x^2+y^2-z}\,dx + \frac{2y}{x^2+y^2-z}\,dy - \frac{1}{x^2+y^2-z}\,dz}$$

46. $u(x,y,z) = \frac{x}{y} + \frac{y}{z} + \frac{z}{x}$, $M(1,2,3)$

$u_x = \frac{1}{y} - \frac{z}{x^2} = \frac{1}{2} - \frac{3}{1} = \frac{1}{2} - \frac{6}{2} = -\frac{5}{2}$

$u_y = -\frac{x}{y^2} + \frac{1}{z} = -\frac{1}{4} + \frac{1}{3} = -\frac{3}{12} + \frac{4}{12} = \frac{1}{12}$

$u_z = -\frac{y}{z^2} + \frac{1}{x} = -\frac{2}{9} + 1 = -\frac{2}{9} + \frac{9}{9} = \frac{7}{9}$

$$\boxed{du = -\frac{5}{2}dx + \frac{1}{12}dy + \frac{7}{9}dz}$$

[redacted] $u(x,y,z) = \sqrt{1-x^2-y^2-z^2}$

45. $u_x = \frac{1}{2\sqrt{1-x^2-y^2-z^2}}\cdot(-2x) = -\frac{x}{\sqrt{1-x^2-y^2-z^2}}$

$u_y = -\frac{y}{\sqrt{1-x^2-y^2-z^2}}$

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$u_z = -\frac{z}{\sqrt{1-x^2-y^2-z^2}}$

$$\boxed{du = -\frac{x}{\sqrt{1-x^2-y^2-z^2}}dx - \frac{y}{\sqrt{1-x^2-y^2-z^2}}dy - \frac{z}{\sqrt{1-x^2-y^2-z^2}}dz}$$

(48) $u(x,y,z)=\ln(xy^2z^3)$; $M(1,1,1)$

$$u_x=\frac{1}{\cancel{xy^2z^3}}\cdot \cancel{y^2z^3}=\frac{1}{x}=1$$

$$u_y=\frac{1}{\cancel{xy^2z^3}}\cdot \cancel{xz^3}\cdot 2y=\frac{2}{y}=2$$

$$u_z=\frac{1}{\cancel{xy^2z^3}}\cdot \cancel{xy^2}\cdot 3z^2=\frac{3}{z}=3$$

$$\boxed{du=dx+2dy+3dz}$$

(49) $u(x,y,z)=2x^2+y^2-z^2-xz+5yz$

$$u_x=4x-z$$

$$u_y=2y+5z$$

$$u_z=-2z-x+5y$$

$$\boxed{du=(4x-z)dx+(2y+5z)dy-(2z+x-5y)dz}$$

(50.) $u(x,y,z)=x^3-\frac{x}{2y}+\frac{y}{z^2-xy}$

$$u_x=3x^2-\frac{1}{2y}-\frac{y}{(z^2-xy)^2}\cdot(-y)=3x^2-\frac{1}{2y}+\frac{y^2}{(z^2-xy)^2}$$

$$u_y=\frac{x}{2y^2}+\frac{z^2\cancel{-xy+xy}}{(z^2-xy)^2}=\frac{x}{2y^2}+\frac{z^2}{(z^2-xy)^2}$$

$$u_z=-\frac{y}{(z^2-xy)^2}\cdot 2z=-\frac{2yz}{(z^2-xy)^2}$$

$$\boxed{du=\left(3x^2-\frac{1}{2y}+\frac{y^2}{(z^2-xy)^2}\right)dx+\left(\frac{x}{2y^2}+\frac{z^2}{(z^2-xy)^2}\right)dy-\frac{2yz}{(z^2-xy)^2}dz}$$

(51.) $u(x,y,z)=\ln(6-2x^2-3y^2-6z^2)$

$$u_x=\frac{-4x}{6-2x^2-3y^2-6z^2}$$

$$u_y=\frac{-6y}{6-2x^2-3y^2-6z^2}$$

$$u_z=\frac{-12z}{6-2x^2-3y^2-6z^2}$$

$$\boxed{du=\frac{-4x}{6-2x^2-3y^2-6z^2}dx-\frac{6y}{6-2x^2-3y^2-6z^2}dy-\frac{12z}{6-2x^2-3y^2-6z^2}dz}$$

(52) $u(x,y,z)=\sqrt{x^2+y^2-z^2}$; $M(1,1,1)$

$$u_x=\frac{2x}{2\sqrt{x^2+y^2-z^2}}=\frac{x}{\sqrt{x^2+y^2-z^2}}=1$$

$$u_y=\frac{y}{\sqrt{x^2+y^2-z^2}}=1$$

$$u_z=\frac{-z}{\sqrt{x^2+y^2-z^2}}=-1$$

$$\boxed{du=dx+dy-dz}$$

(53.) $u(x,y,z)=\dfrac{y\sin x}{\cos(z-x)}$

$$u_x=\frac{y\cos x\cdot\cos(z-x)-y\sin x\cdot\sin(z-x)}{\cos^2(z-x)}$$

$$u_y=\frac{\sin x}{\cos(z-x)}$$

$$u_z=-\frac{y\sin x}{\cos^2(z-x)}\cdot(-\sin(z-x))\cdot 1=\frac{y\sin x\sin(z-x)}{\cos^2(z-x)}$$

$$\boxed{du=\frac{y(\cos x\cdot\cos(z-x)-\sin x\cdot\sin(z-x))}{\cos^2(z-x)}dx+\frac{\sin x}{\cos(z-x)}dy+\frac{y\sin x\sin(z-x)}{\cos^2(z-x)}dz}$$

(54.) $u(x,y,z)=\dfrac{x\,\mathrm{ch}\,z}{\mathrm{sh}(z-y)}$

$$u_x=\frac{\mathrm{ch}\,z}{\mathrm{sh}(z-y)}$$

$$u_y=-\frac{x\,\mathrm{ch}\,z}{\mathrm{sh}^2(z-y)}\cdot\mathrm{ch}(z-y)\cdot(-1)=\frac{x\,\mathrm{ch}\,z\cdot\mathrm{ch}(z-y)}{\mathrm{sh}^2(z-y)}$$

$$u_z=\frac{x\,\mathrm{sh}\,z\cdot\mathrm{sh}(z-y)-x\,\mathrm{ch}\,z\cdot\mathrm{ch}(z-y)}{\mathrm{sh}^2(z-y)}$$

$$\boxed{du=\frac{\mathrm{ch}\,z}{\mathrm{sh}(z-y)}dx+\frac{x\,\mathrm{ch}\,z\,\mathrm{ch}(z-y)}{\mathrm{sh}^2(z-y)}dy+\frac{x(\mathrm{sh}\,z\,\mathrm{sh}(z-y)-\mathrm{ch}\,z\,\mathrm{ch}(z-y))}{\mathrm{sh}^2(z-y)}dz}$$

(26) $z(x,y)=\dfrac{x}{y}\,\mathrm{sh}\dfrac{y}{x}$

$$z_x=\frac{1}{y}\mathrm{sh}\frac{y}{x}+\frac{\cancel{x}}{\cancel{y}}\cdot\mathrm{ch}\frac{y}{x}\cdot\left(-\frac{\cancel{y}}{x^{\cancel{2}}}\right)=\frac{1}{y}\mathrm{sh}\frac{y}{x}-\frac{1}{x}\mathrm{ch}\frac{y}{x}$$

$$z_y=-\frac{x}{y^2}\mathrm{sh}\frac{y}{x}+\frac{\cancel{x}}{y}\cdot\mathrm{ch}\frac{y}{x}\cdot\frac{1}{\cancel{x}}=\frac{1}{y}\mathrm{ch}\frac{y}{x}-\frac{x}{y^2}\mathrm{sh}\frac{y}{x}$$

$$\boxed{dz=\left(\frac{1}{y}\mathrm{sh}\frac{y}{x}-\frac{1}{x}\mathrm{ch}\frac{y}{x}\right)dx+\frac{1}{y}\left(\mathrm{ch}\frac{y}{x}-\frac{x}{y}\mathrm{sh}\frac{y}{x}\right)dy}$$

(27.) $z(x,y)=\frac{x}{y^2}+y\,sh^2x$

$z_x=\frac{1}{y^2}+y\cdot 2shx\cdot chx$

$\boxed{2shxchx = sh2x\ ?}$???

$z_y=-\frac{2x}{y^3}+sh^2x$

$$\boxed{dz=\left(\frac{1}{y^2}+y\cdot 2shx\,chx\right)dx+\left(sh^2x-\frac{2x}{y^3}\right)dy}$$

(28.) $z(x,y)=\frac{x^2}{y^3}+x\,ch^2y$

$z_x=\frac{2x}{y^3}+ch^2y$

$z_y=-\frac{3x^2}{y^4}+x\cdot 2chy\cdot shy$

$$\boxed{dz=\left(\frac{2x}{y^3}+ch^2y\right)dx+\left(x\cdot 2chy\,shy-\frac{3x^2}{y^4}\right)dy}$$

$\Rightarrow$ ~~28~~, 29 ?

ПОГЛАВЉЕ 1.3

(1.) $z(x,y)=x^4-2xy-y^3$

$z_x=4x^3-2y$

$z_{xx}=12x^2$

$z_{xy}=-2$

$z_y=-2x-3y^2$

$z_{yy}=-6y$

$$\boxed{d^2z=12x^2dx^2-4dxdy-6ydy^2}$$ ✓

$\left(\frac{1}{y^2}\right)'=-\frac{1}{\ }$

$\frac{1}{x}=-\frac{1}{x^2}$

$\frac{1}{x^2}=-\frac{2}{x^3}$

$\frac{1}{x^3}=\frac{3}{x^4}$ $\quad \frac{1}{x^2}=\frac{-2x}{x^4}=-\frac{2}{x^3}$

(2.) $z(x,y)=\frac{2x}{x-y^2}$

$z_x=\frac{2(x-y^2)-2x}{(x-y^2)^2}=\frac{\cancel{2x}-2y^2-\cancel{2x}}{(x-y^2)^2}=\frac{-2y^2}{(x-y^2)^2}$

$z_{xx}=\frac{-4y^2}{(x-y^2)^3}$

$z_{xy}=\frac{-4y(x-y^2)^2+2y^2\cdot 2(x-y^2)\cdot(-2y)}{(x-y^2)^4}=\frac{-4y\cancel{(x-y^2)}(x-y^2+2y^2)}{(x-y^2)^{\cancel{4}\,3}}$

$$= -\frac{4y(x+y^2)}{(x-y^2)^3}$$

$$z_y = -\frac{2x}{(x-y^2)^2}\cdot(-2y) = \frac{4xy}{(x-y^2)^2}$$

$$z_{yy} = \frac{4x(x-y^2)^2 - 4xy\cdot 2(x-y^2)(-2y)}{(x-y^2)^4} = \frac{4x\cancel{(x-y^2)}(x-y^2+4y^2)}{(x-y^2)^{\cancel{4}\,3}} = \frac{4x(x+3y^2)}{(x-y^2)^3}$$

$$\boxed{d^2z = -\frac{4y^2}{(x-y^2)^3}dx^2 - \frac{8y(x+y^2)}{(x-y^2)^3}dx\,dy + \frac{4x(x+3y^2)}{(x-y^2)^3}dy^2}$$

3. $z(x,y) = \dfrac{x-y}{x+y}$

$$z_x = \frac{\cancel{x}+y-\cancel{x}+y}{(x+y)^2} = \frac{2y}{(x+y)^2}$$

$$z_{xx} = -\frac{4y}{(x+y)^3}$$

$$z_{xy} = \frac{2(x+y)^2 - 2y\cdot 2(x+y)}{(x+y)^4} = \frac{2\cancel{(x+y)}(x+y-2y)}{(x+y)^{\cancel{4}\,3}} = \frac{2(x-y)}{(x+y)^3}$$

$$z_y = \frac{-x-\cancel{y}-x+\cancel{y}}{(x+y)^2} = \frac{-2x}{(x+y)^2}$$

$$z_{yy} = \frac{4x}{(x+y)^3}$$

$$\boxed{d^2z = -\frac{4y}{(x+y)^3}d^2x + \frac{4(x-y)}{(x+y)^3}dx\,dy + \frac{4x}{(x+y)^3}d^2y}$$

4.* $z(x,y) = x\sqrt{x+y^2}$; $M(3,1)$

$$z_x = \sqrt{x+y^2} + x\frac{1}{2\sqrt{x+y^2}} = \sqrt{x+y^2} + \frac{x}{2\sqrt{x+y^2}} = \frac{2(x+y^2)+x}{2\sqrt{x+y^2}} = \frac{3x+2y^2}{2\sqrt{x+y^2}}$$

$$z_{xx} = \frac{6\sqrt{x+y^2} - (3x+2y^2)\cdot\frac{1}{\sqrt{x+y^2}}}{4(x+y^2)} = \frac{6(x+y^2)-3x-2y^2}{4(x+y^2)\sqrt{x+y^2}} =$$

$$= \frac{6x+6y^2-3x-2y^2}{4(x+y^2)\sqrt{x+y^2}} = \frac{3x+4y^2}{4(x+y^2)^{3/2}} = \frac{9+4}{32} = \frac{13}{32}$$

$$z_{xy} = \frac{4y\cdot 2\sqrt{x+y^2} - (3x+2y^2)\cdot\frac{1}{\sqrt{x+y^2}}\cdot 2y}{4(x+y^2)} =$$

$$= \frac{8y(x+y^2) - 2y(3x+2y^2)}{4(x+y^2)^{3/2}} = \frac{8xy+8y^3-6xy-4y^3}{4(x+y^2)^{3/2}} = \frac{\cancel{2}xy+\cancel{4}^{2}y^3}{2\cancel{4}(x+y^2)^{3/2}} =$$

$$= \frac{y(x+2y^2)}{2(x+y^2)^{3/2}} = \frac{5}{16}$$

$$z_y = \frac{x}{\cancel{2}\sqrt{x+y^2}} \cdot \cancel{2}y = \frac{xy}{\sqrt{x+y^2}}$$

$$z_{yy} = \frac{x\sqrt{x+y^2} - xy \cdot \frac{1}{\cancel{2}\sqrt{x+y^2}} \cdot \cancel{2}y}{x+y^2} = \frac{x\sqrt{x+y^2} - \frac{xy^2}{\sqrt{x+y^2}}}{x+y^2} =$$

$$= \frac{x(x+y^2) - xy^2}{(x+y^2)^{3/2}} = \frac{x^2 + \cancel{xy^2} - \cancel{xy^2}}{(x+y^2)^{3/2}} = \frac{x^2}{(x+y^2)^{3/2}} = \frac{9}{8}$$

$$\boxed{d^2z = \frac{13}{32}d^2x + \frac{5}{8}dxdy + \frac{9}{8}d^2y}$$

(5) $z(x,y) = xy + \sqrt{y}$

$z_x = y$

$z_{xx} = 0$

$z_{xy} = 1$

$z_y = x + \frac{1}{2\sqrt{y}}$

$z_{yy} = -\frac{1}{2y} \cdot \frac{1}{2\sqrt{y}} = -\frac{1}{4y^{3/2}}$

$$\boxed{d^2z = 2dxdy - \frac{1}{4\sqrt{y^3}}d^2y}$$ ✓

(6) $z(x,y) = \frac{x}{y} + \frac{y}{x}$

$z_x = \frac{1}{y} - \frac{y}{x^2}$

$z_{xx} = +\frac{2y}{x^3}$

$z_{xy} = -\frac{1}{y^2} - \frac{1}{x^2}$

$z_y = -\frac{x}{y^2} + \frac{1}{x}$

$z_{yy} = +\frac{2x}{y^3}$

$$\boxed{d^2z = +\frac{2y}{x^3}d^2x - 2\left(\frac{1}{y^2} + \frac{1}{x^2}\right)dxdy + \frac{2x}{y^3}d^2y}$$ ✓

(7) $z(x,y) = x\sin(x+y)$

$z_x = \sin(x+y) + x\cos(x+y)$

$z_{xx} = \cos(x+y) + \cos(x+y) - x\sin(x+y) = 2\cos(x+y) - x\sin(x+y)$

$z_{xy} = \cos(x+y) - x\sin(x+y)$

$z_y = x\cos(x+y)$

$z_{yy} = -x\sin(x+y)$

$$\boxed{d^2z = \left(2\cos(x+y) - x\sin(x+y)\right)d^2x + 2\left(\cos(x+y) - x\sin(x+y)\right)dx\,dy - x\sin(x+y)\,d^2y}$$

8) $z(x,y) = \ln(x+y^2)$; $M(1,1)$

$z_x = \frac{1}{x+y^2}$

$z_{xx} = -\frac{1}{(x+y^2)^2} = -\frac{1}{4}$

$z_{xy} = -\frac{1}{(x+y^2)^2}\cdot 2y = -\frac{1}{2}$

$z_y = \frac{2y}{x+y^2}$

$z_{yy} = \frac{2(x+y^2) - 2y\cdot 2y}{(x+y^2)^2} = \frac{2x+2y^2-4y^2}{(x+y^2)^2} = \frac{2x-2y^2}{(x+y^2)^2} = 0$

$$\boxed{d^2z = -\frac{1}{4}d^2x - dx\,dy}$$

9) $z(x,y) = \sqrt{xy}$; $M(2,2)$

$z_x = \frac{1}{2\sqrt{xy}}\cdot y = \frac{y}{2\sqrt{xy}} = \frac{y}{2}\cdot\frac{1}{\sqrt{xy}}$

$z_{xx} = \frac{y}{2}\left(-\frac{1}{xy}\cdot\frac{1}{2\sqrt{xy}}\cdot y\right) = -\frac{y}{4x\sqrt{xy}} = -\frac{2}{8\cdot 2} = -\frac{1}{8}$

$z_{xy} = \frac{2\sqrt{xy} - y\cdot 2\cdot\frac{1}{2\sqrt{xy}}\cdot x}{4xy} = \frac{2xy - xy}{4xy\sqrt{xy}} = \frac{xy}{4xy\sqrt{xy}} = \frac{1}{4\sqrt{xy}} = \frac{1}{8}$

$z_y = \frac{1}{2\sqrt{xy}}\cdot x = \frac{x}{2\sqrt{xy}}$

$z_{yy} = -\frac{x}{4xy}\cdot 2\cdot\frac{1}{2\sqrt{xy}}\cdot x = -\frac{x^2}{4xy\sqrt{xy}} = -\frac{x}{4y\sqrt{xy}} = -\frac{1}{8}$

$$\boxed{d^2z = -\frac{1}{8}d^2x + \frac{1}{4}dx\,dy - \frac{1}{8}d^2y}$$

(10.) $z(x,y) = y\ln(x^2+y^2)$; $M(0,1)$

$$Z_x = \frac{2xy}{x^2+y^2}$$

$$Z_{xx} = \frac{2y(x^2+y^2) - 2xy\cdot 2x}{(x^2+y^2)^2} = \frac{2x^2y + 2y^3 - 4x^2y}{(x^2+y^2)^2} = \frac{2y^3-2x^2y}{(x^2+y^2)^2} = \frac{2y(y^2-x^2)}{(x^2+y^2)^2} = 2$$

$$Z_{xy} = \frac{2x(x^2+y^2) - 2xy\cdot 2y}{(x^2+y^2)^2} = \frac{2x^3+2xy^2-4xy^2}{(x^2+y^2)^2} = \frac{2x^3-2xy^2}{(x^2+y^2)^2} = \frac{2x(x^2-y^2)}{(x^2+y^2)^2} = 0$$

$$Z_y = \ln(x^2+y^2) + y\cdot\frac{1}{x^2+y^2}\cdot 2y = \ln(x^2+y^2) + \frac{2y^2}{x^2+y^2}$$

$$Z_{yy} = \frac{2y}{x^2+y^2} + \frac{4y(x^2+y^2) - 2y^2\cdot 2y}{(x^2+y^2)^2} = \frac{2y(x^2+y^2) + 4x^2y + \cancel{4y^3} - \cancel{4y^3}}{(x^2+y^2)^2} =$$

$$= \frac{2x^2y + 2y^3 + 4x^2y}{(x^2+y^2)^2} = \frac{2y^3+6x^2y}{(x^2+y^2)^2} = \frac{2y(y^2+3x^2)}{(x^2+y^2)^2} = 2$$

$$\boxed{d^2z = 2d^2x + 2d^2y}$$

(11.) $z(x,y) = xy\ln(xy)$; $M(1,e)$

$$Z_x = y\ln(xy) + \cancel{xy}\cdot\frac{1}{\cancel{xy}}\cdot y = y\ln(xy) + y$$

$$Z_{xx} = \frac{y^{\cancel{2}}}{\cancel{x}y} = \frac{y}{x} = e$$

$$Z_{xy} = \ln(xy) + \cancel{y}\cdot\frac{1}{\cancel{xy}}\cdot\cancel{x} + 1 = \ln(xy) + 1 = 1+1 = 2$$

$$Z_y = x\ln(xy) + \cancel{xy}\cdot\frac{1}{\cancel{xy}}\cdot x = x\ln(xy) + x$$

$$Z_{yy} = \frac{x^{\cancel{2}}}{\cancel{x}y} = \frac{x}{y} = \frac{1}{e}$$

$$\boxed{d^2z = e\cdot d^2x + 4dx\,dy + \frac{1}{e}\cdot d^2y}$$

(12.) $z(x,y) = (2x^4+3y^2)^5$; $M(0,1)$

$$Z_x = 5(2x^4+3y^2)^4\cdot 8x^3 = 40x^3(2x^4+3y^2)^4$$

$$Z_{xx} = 40\cdot 3x^2(2x^4+3y^2)^4 + 40x^3\cdot 4(2x^4+3y^2)^3\cdot 8x^3 =$$

$$= 40x^2(2x^4+3y^2)^3\left(3(2x^4+3y^2) + 4x\cdot 8x^3\right) =$$

$$= 40x^2(2x^4+3y^2)^3(8x^4+9y^2+32x^4) =$$

$$= 40x^2(2x^4+3y^2)^3(40x^4+9y^2) = 0$$

$$Z_{xy} = 40x^3\cdot 4(2x^4+3y^2)^3\cdot 6y = 0$$

$$Z_y = 5(2x^4+3y^2)^4\cdot 6y = 30y(2x^4+3y^2)^4$$

$$Z_{yy} = 30(2x^4+3y^2)^4 + 30y\cdot 4(2x^4+3y^2)^3\cdot 6y =$$

$$= 30(2x^4+3y^2)^3(2x^4+3y^2+24y^2) = 30(2x^4+3y^2)^3(2x^4+27y^2) = 21870$$

$$\boxed{d^2z = 21870\,d^2y}$$

13. $z(x,y) = \operatorname{arctg}\frac{x}{y}$

$$z_x = \frac{1}{1+\left(\frac{x}{y}\right)^2}\cdot\frac{1}{y} = \frac{y^{\cancel{2}}}{x^2+y^2}\cdot\frac{1}{\cancel{y}} = \frac{y}{x^2+y^2}$$

$$z_{xx} = -\frac{y}{(x^2+y^2)^2}\cdot 2x = -\frac{2xy}{(x^2+y^2)^2}$$

$$z_{xy} = \frac{x^2+y^2-y\cdot 2y}{(x^2+y^2)^2} = \frac{x^2+y^2-2y^2}{(x^2+y^2)^2} = \frac{x^2-y^2}{(x^2+y^2)^2}$$

$$z_y = \frac{1}{1+\left(\frac{x}{y}\right)^2}\cdot\left(-\frac{x}{y^2}\right) = -\frac{\cancel{y^2}}{x^2+y^2}\cdot\frac{x}{\cancel{y^2}} = -\frac{x}{x^2+y^2}$$

$$z_{yy} = \frac{x}{(x^2+y^2)^2}\cdot 2y = \frac{2xy}{(x^2+y^2)^2}$$

$$\boxed{d^2z = -\frac{2xy}{(x^2+y^2)^2}d^2x + \frac{2(x^2-y^2)}{(x^2+y^2)^2}dx\,dy + \frac{2xy}{x^2+y^2}d^2y}$$

14. $z(x,y) = \operatorname{arctg}\frac{y}{x}$, $M(1,1)$

$$z_x = \frac{1}{1+\left(\frac{y}{x}\right)^2}\cdot\left(-\frac{y}{x^2}\right) = -\frac{\cancel{x^2}}{x^2+y^2}\cdot\frac{y}{\cancel{x^2}} = -\frac{y}{x^2+y^2}$$

$$z_{xx} = \frac{y}{(x^2+y^2)^2}\cdot 2x = \frac{2xy}{(x^2+y^2)^2} = \frac{1}{2}$$

$$z_{xy} = \frac{-x^2-y^2+y\cdot 2y}{(x^2+y^2)^2} = \frac{y^2-x^2}{(x^2+y^2)^2} = 0$$

$$z_y = \frac{1}{1+\left(\frac{y}{x}\right)^2}\cdot\frac{1}{x} = \frac{x^{\cancel{2}}}{x^2+y^2}\cdot\frac{1}{\cancel{x}} = \frac{x}{x^2+y^2}$$

$$z_{yy} = -\frac{x}{(x^2+y^2)^2}\cdot 2y = -\frac{2xy}{(x^2+y^2)^2} = -\frac{1}{2}$$

$$\boxed{d^2z = \frac{1}{2}d^2x - \frac{1}{2}d^2y}$$

15. $z(x,y) = x\arcsin y$

$$z_x = \arcsin y$$

$$z_{xx} = 0$$

$$z_{xy} = \frac{1}{\sqrt{1-y^2}}$$

$$\boxed{d^2z = \frac{2}{\sqrt{1-y^2}}dx\,dy + \frac{xy}{(1-y^2)^{3/2}}d^2y}$$

$$z_y = \frac{x}{\sqrt{1-y^2}}$$

$$z_{yy} = -\frac{x}{1-y^2}\cdot\frac{1}{2\sqrt{1-y^2}}\cdot(-2y) = \frac{\cancel{2}xy}{\cancel{2}(1-y^2)^{3/2}} = \frac{xy}{(1-y^2)^{3/2}}$$

(16) $z(x,y) = y \operatorname{arctg} x$; $M(1,-1)$

$$z_x = y \cdot \frac{1}{1+x^2} = \frac{y}{1+x^2} \checkmark$$

$$z_{xx} = -\frac{y}{(1+x^2)^2} \cdot 2x = -\frac{2xy}{(1+x^2)^2} = \frac{1}{2} \checkmark$$

$$z_{xy} = \frac{1}{1+x^2} = \frac{1}{2} \checkmark$$

$$z_y = \operatorname{arctg} x \checkmark$$

$$z_{yy} = 0 \checkmark$$

$$\boxed{d^2z = \frac{1}{2} d^2x + dx\,dy}$$

(17) $z(x,y) = \sqrt{2x^2+3y^2-4}$; $M(2,0)$

$$z_x = \frac{4x}{2\sqrt{2x^2+3y^2-4}} = \frac{2x}{\sqrt{2x^2+3y^2-4}} \checkmark$$

$$z_{xx} = \frac{2\sqrt{2x^2+3y^2-4} - 2x \cdot \frac{2x}{\sqrt{2x^2+3y^2-4}}}{2x^2+3y^2-4} =$$

$$= \frac{2(2x^2+3y^2-4) - 4x^2}{(2x^2+3y^2-4)^{3/2}} = \frac{\cancel{4x^2}+6y^2-8-\cancel{4x^2}}{(2x^2+3y^2-4)^{3/2}} = \frac{6y-8}{(2x^2+3y^2-4)^{3/2}} = -\frac{8}{8} = -1$$

$$z_{xy} = -\frac{\cancel{2x}}{2x^2+3y^2-4} \cdot \frac{1}{\cancel{2}\sqrt{2x^2+3y^2-4}} \cdot 6y =$$

$$= -\frac{6xy}{(2x^2+3y^2-4)^{3/2}} = 0$$

$$z_y = \frac{3y}{\sqrt{2x^2+3y^2-4}}$$

$$z_{yy} = \frac{3\sqrt{2x^2+3y^2-4} - 3y \cdot \frac{1}{\cancel{2}\sqrt{2x^2+3y^2-4}} \cdot \overset{3}{\cancel{6}}y}{2x^2+3y^2-4} =$$

$$= \frac{3(2x^2+3y^2-4) - 9y^2}{(2x^2+3y^2-4)^{3/2}} = \frac{6x^2+\cancel{9y^2}-12-\cancel{9y^2}}{(2x^2+3y^2-4)^{3/2}} = \frac{6x^2-12}{(2x^2+3y^2-4)^{3/2}} = \frac{12}{8} = \frac{3}{2}$$

$$\boxed{d^2z = -d^2x + \frac{3}{2} d^2y}$$

(18) $z(x,y) = x^3 + 2xy - y^2 - 2x + y$

$z_x = 3x^2 + 2y - 2$

$z_{xx} = 6x$

$z_{xy} = 2$

$z_y = 2x - 2y + 1$

$z_{yy} = -2$

$$\boxed{d^2z = 6x\,d^2x + 4dxdy - 2d^2y}$$

(19) $z(x,y) = x^5 - 3xy - y^3 + 3x, \quad M(0,1)$

$z_x = 5x^4 - 3y + 3$

$z_{xx} = 20x^3 = 0$

$z_{xy} = -3$

$z_y = -3x - 3y^2$

$z_{yy} = -6y = -6$

$$\boxed{d^2z = -6dxdy - 6d^2y}$$

(20) $z(x,y) = xy(2 - x^2 - y)$

$z_x = y(2-x^2-y) + xy\cdot(-2x) = 2y - x^2y - y^2 - 2x^2y = 2y - y^2 - 3x^2y$

$z_{xx} = -6xy$

$z_{xy} = 2 - 2y = 2(1-y)$ $\quad 2 - 2y - 3x^2$ ✓

$z_y = x(2-x^2-y) + xy(-1) = 2x - x^3 - xy - xy = 2x - x^3 - 2xy$ ✓

$z_{yy} = -2x$ ✓

$$\boxed{d^2z = -6xy\cdot d^2x + 4(1-y)dxdy - 2x\,d^2y}$$

(21) $z(x,y) = (x^2+y^2)e^{x-y}; \quad M(1,1)$

$z_x = 2x\cdot e^{x-y} + (x^2+y^2)\cdot e^{x-y} = e^{x-y}(2x + x^2 + y^2)$

$z_{xx} = e^{x-y}(2x+x^2+y^2) + e^{x-y}\cdot(2+2x) = e^{x-y}(2x+x^2+y^2+2+2x) = e^{x-y}(4x+x^2+y^2+2) = 8$

$z_{xy} = -e^{x-y}(2x+x^2+y^2) + e^{x-y}\cdot 2y = e^{x-y}(2y - 2x - x^2 - y^2) = -2$

$z_y = 2y\cdot e^{x-y} + (x^2+y^2)\cdot e^{x-y}(-1) = e^{x-y}(2y - x^2 - y^2)$

$z_{yy} = -e^{x-y}(2y-x^2-y^2) + e^{x-y}\cdot(2-2y) = e^{x-y}(2-2y-2y+x^2+y^2) = e^{x-y}(2-4y+x^2+y^2) = 0$

$$\boxed{d^2z = 8d^2x - 4dxdy}$$

(22.) $z(x,y) = (y^2-2x)e^{y-x}$

$Zx = -2\cdot e^{y-x} + (y^2-2x)\cdot e^{y-x}(-1) = e^{y-x}(-2-y^2+2x)$

$Zxx = -e^{y-x}(-2-y^2+2x) + e^{y-x}\cdot 2 = e^{y-x}(2+2+y^2-2x) = e^{y-x}(4+y^2-2x)$

$Zxy = e^{y-x}(-2-y^2+2x) + e^{y-x}\cdot(-2y) = e^{y-x}(2x-2y-y^2-2)$

$Zy = 2y\cdot e^{y-x} + (y^2-2x)\cdot e^{y-x} = e^{y-x}(2y-2x+y^2)$

$Zyy = e^{y-x}(2y-2x+y^2) + e^{y-x}(2+2y) = e^{y-x}(4y-2x+y^2+2)$

$$\boxed{d^2z = e^{y-x}(4+y^2-2x)d^2x + 2\cdot e^{y-x}(2x-2y-y^2-2)dxdy + e^{y-x}(4y-2x+y^2+2)d^2y}$$

(23.) $z(x,y) = e^{xy^2}$; $M(1,1)$

$Zx = e^{xy^2}\cdot y^2$

$Zxx = e^{xy^2}\cdot y^4 = e$

$Zxy = e^{xy^2}\cdot 2xy\cdot y^2 + e^{xy^2}\cdot 2y = e^{xy^2}(2y+2xy^3) = 4e$

$Zy = e^{xy^2}\cdot 2xy$

$Zyy = e^{xy^2}\cdot 2y\cdot 2xy + e^{xy^2}\cdot 2x = e^{xy^2}(2x+4xy^2) = 6e$

$$\boxed{d^2z = e\cdot d^2x + 8e\,dxdy + 6e\,d^2y}$$

(24.) $z(x,y) = y\cdot e^{xy}$

$Zx = y\,e^{xy}\cdot y = y^2\cdot e^{xy}$

$Zxx = y^2\cdot e^{xy}\,y = y^3\cdot e^{xy}$

$Zxy = 2y\cdot e^{xy} + y^2\cdot e^{xy}\cdot x = e^{xy}(2y+xy^2)$

$Zy = e^{xy} + y\cdot e^{xy}\cdot x = e^{xy}(xy+1)$

$Zyy = e^{xy}\cdot x(xy+1) + e^{xy}\cdot x = xe^{xy}(xy+2)$

$$\boxed{d^2z = y^3\cdot e^{xy}d^2x + 2e^{xy}(2y+xy^2)dxdy + xe^{xy}(xy+2)\,d^2y}$$

~~(25.) (26.) → sh, ch M(0,1)~~

(27.) $z(x,y) = \frac{x}{y} + y\,sh^2x$ $\boxed{sh^2x + ch^2x = 1}$?!

$Zx = \frac{1}{y} + y\cdot 2shx\cdot chx$

$Zxx = 2y\,ch^2x + 2y\,sh^2x = 2y(sh^2x + ch^2x)$

$Zxy = -\frac{1}{y^2} + 2shx\cdot chx$

$Zy = -\frac{x}{y^2} + sh^2x$

$$z_{yy} = \frac{2x}{y^3}$$

$$\boxed{d^2z = 2y(sh^2x + ch^2x)d^2x + 2\left(2sh\,x\,ch\,x - \frac{1}{y^2}\right)dxdy + \frac{2x}{y^3}d^2y}$$

(28.) $z(x,y) = \frac{x^2}{y^3} + x\,ch^2y$

$$z_x = \frac{2x}{y^3} + ch^2y$$

$$z_{xx} = \frac{2}{y^3}$$

$$z_{xy} = -\frac{6x}{y^4} + 2ch\,y\,sh\,y$$

$$z_y = -\frac{3x^2}{y^4} + x \cdot 2ch\,y\,sh\,y$$

$$z_{yy} = \frac{12x^2}{y^5} + 2x\,sh^2y + 2x\,ch^2y = \frac{12x^2}{y^5} + 2x(sh^2y + ch^2y)$$

$$\boxed{d^2z = \frac{2}{y^3}d^2x + 2\left(2ch\,y\,sh\,y - \frac{6x}{y^4}\right)dxdy + \left(\frac{12x^2}{y^5} + 2x(sh^2y + ch^2y)\right)d^2y}$$

(29.) $z(x,y) = \sqrt[3]{(1-2x+3y)^2}$; $M(0,0)$

$$z_x = \frac{1}{3\cdot\sqrt[3]{(1-2x+3y)^4}} \cdot 2(1-2x+3y)(-2) = \frac{-4(1-2x+3y)}{3\cdot\sqrt[3]{(1-2x+3y)^4}} = -\frac{4}{3\cdot\sqrt[3]{1-2x+3y}}$$

$$z_{xx} = -\frac{4}{3}\cdot\left(-\frac{1}{\sqrt[3]{(1-2x+3y)^2}}\right)\cdot\frac{1}{3\cdot\sqrt[3]{(1-2x+3y)^2}}\cdot(-2) =$$

$$= -\frac{8}{3}\cdot\frac{1}{\sqrt[3]{(1-2x+3y)^4}} = -\frac{8}{3}$$

$$z_{xy} = -\frac{4}{3}\left(-\frac{1}{\cancel{3}}(1-2x+3y)^{-\frac{4}{3}}\cdot\cancel{3}\right) = \frac{4}{3}\cdot(1-2x+3y)^{-\frac{4}{3}} = \frac{4}{3}$$

$$z_y = \frac{1}{\cancel{3}\cdot\sqrt[3]{(1-2x+3y)^4}}\cdot 2(1-2x+3y)\cdot\cancel{3} =$$

$$= \frac{2(1-2x+3y)}{\sqrt[3]{(1-2x+3y)^4}} = \frac{2}{\sqrt[3]{1-2x+3y}}$$

$$z_{yy} = -\frac{2}{\sqrt[3]{(1-2x+3y)^2}}\cdot\frac{1}{\cancel{3}\cdot\sqrt[3]{(1-2x+3y)^2}}\cdot\cancel{3} = -\frac{2}{\sqrt[3]{(1-2x+3y)^4}} = -2$$

$$\boxed{d^2z = -\frac{8}{3}d^2x + \frac{8}{3}dxdy - 2d^2y}$$

(30.) $z(x,y) = \sqrt[5]{(1-x+4y)^4}$; $M(0,0)$

$z_x = \frac{4}{5}(1-x+4y)^{-\frac{1}{5}}(-1) = -\frac{4}{5}(1-x+4y)^{-\frac{1}{5}}$

$z_{xx} = -\frac{4}{5}\left(-\frac{1}{5}(1-x+4y)^{-\frac{6}{5}}(-1)\right) = -\frac{4}{25}(1-x+4y)^{-\frac{6}{5}} = -\frac{4}{25}$

$z_{xy} = -\frac{4}{5}\cdot\left(-\frac{1}{5}(1-x+4y)^{-\frac{6}{5}}\cdot 4\right) = \frac{16}{25}(1-x+4y)^{-\frac{6}{5}} = \frac{16}{25}$

$z_y = \frac{4}{5}(1-x+4y)^{-\frac{1}{5}}\cdot 4 = \frac{16}{5}(1-x+4y)^{-\frac{1}{5}}$

$z_{yy} = \frac{16}{5}\left(-\frac{1}{5}(1-x+4y)^{-\frac{6}{5}}\cdot 4\right) = -\frac{64}{25}(1-x+4y)^{-\frac{6}{5}} = -\frac{64}{25}$

$$\boxed{d^2z = -\frac{4}{25}d^2x + \frac{32}{25}dx\,dy - \frac{64}{25}d^2y}$$

(31) $z(x,y) = y\sin(x+2y)$; $M(0, \pi/4)$

$z_x = y\cdot\cos(x+2y)$

$z_{xx} = -y\cdot\sin(x+2y) = -\pi/4$

$z_{xy} = \cos(x+2y) - y\sin(x+2y)\cdot 2 = -\pi/2$

$z_y = \sin(x+2y) + y\cdot\cos(x+2y)\cdot 2$

$z_{yy} = \cos(x+2y)\cdot 2 + \cos(x+2y) - 2y\cdot\sin(x+2y)\cdot 2 =$

$= 3\cdot\cos(x+2y) - 4\cdot y\sin(x+2y) = -\pi$

$$\boxed{d^2z = -\frac{\pi}{4}d^2x - \frac{\pi}{2}dx\,dy - \pi d^2y}$$

(32.) $z(x,y) = \ln\frac{x^2-y^2}{xy}$

$z_x = \frac{\cancel{xy}}{x^2-y^2}\cdot\frac{2x\cdot xy - (x^2-y^2)\cdot y}{(xy)^{\cancel{2}}} = \frac{2x^2y - x^2y + y^3}{xy(x^2-y^2)} = \frac{x^2y+y^3}{xy(x^2-y^2)} = \frac{y(x^2+y^2)}{xy(x^2-y^2)} = \frac{x^2+y^2}{x(x^2-y^2)}$

$z_{xx} = \frac{2x\cdot(x^3-xy^2) - (x^2+y^2)\cdot(3x^2-y^2)}{x^2(x^2-y^2)^2} = \frac{2x^4-2x^2y^2-3x^4+x^2y^2-3x^2y^2+y^4}{x^2(x^2-y^2)^2} =$

$= \frac{-x^4+y^4-4x^2y^2}{x^2(x^2-y^2)^2}$

$z_y = \frac{\cancel{xy}}{x^2-y^2}\cdot\frac{-2y\cdot xy - (x^2-y^2)\cdot x}{(xy)^2} = \frac{-2xy^2-x^3+xy^2}{xy(x^2-y^2)} = \frac{-xy^2-x^3}{xy(x^2-y^2)} = \frac{-x(y^2+x^2)}{\cancel{x}y(x^2-y^2)} = \frac{-y^2-x^2}{y(x^2-y^2)}$

$z_{yy} = \frac{-2y\cdot y(x^2-y^2) - (-y^2-x^2)\cdot\left((x^2-y^2)+y\cdot(-2y)\right)}{y^2(x^2-y^2)^2} =$

$= \frac{-2y^2(x^2-y^2)+(y^2+x^2)(x^2-y^2-2y^2)}{y^2(x^2-y^2)^2} = \frac{-2x^2y^2+2y^4+(y^2+x^2)(x^2-3y^2)}{y^2(x^2-y^2)^2} =$

$= \frac{-2x^2y^2+2y^4+x^2y^2-3y^4+x^4-3x^2y^2}{y^2(x^2-y^2)^2} = \frac{x^4-y^4-4x^2y^2}{y^2(x^2-y^2)^2}$

$z_{yx} = \frac{(4x^3-8xy^2)y^2(x^2-y^2)^2-(x^4-y^4-4x^2y^2)\cdot y^2\cdot 2(x^2-y^2)\cdot 2x}{y^4(x^2-y^2)^4} =$

$$= \frac{\cancel{y^2(x^2-y^2)}\left((x^2-y^2)(4x^3-8xy^2)-4x(x^4-y^4-4x^2y^2)\right)}{y^{4}(x^2-y^2)^{4\,3}} =$$

$$= \frac{\cancel{4x^5}-8x^3y^2-4x^3y^2+8xy^4-\cancel{4x^5}+4xy^4+16x^3y^2}{y^2(x^2-y^2)^3} =$$

$$= \frac{4x^3y^2+12xy^4}{y^2(x^2-y^2)^3} = \frac{4x\cancel{y^2}(x^2+3y^2)}{\cancel{y^2}(x^2-y^2)^3} = \frac{4x(x^2+3y^2)}{(x^2-y^2)^3}$$

$$\boxed{d^2z = \frac{x^4+y^4-4x^2y^2}{x^2(x^2-y^2)^2}d^2x + \frac{8x(x^2+3y^2)}{(x^2-y^2)^3}dxdy + \frac{x^4-y^4-4x^2y^2}{y^2(x^2-y^2)^2}d^2y}$$

(33) $z(x,y) = (x+y^2)\sqrt{e^y} = x\sqrt{e^y} + y^2\sqrt{e^y}$

$$\frac{e^x}{\sqrt{e^x}} \cdot \frac{\sqrt{e^x}}{\sqrt{e^x}} = \frac{e^x\sqrt{e^x}}{e^x} = \sqrt{e^x}$$

$Z_x = \sqrt{e^y}$

$1-\left(-\frac{1}{2}\right) = 1+\frac{1}{2} = \frac{1}{2}$

$Z_{xx} = 0$

$$Z_{xy} = \frac{1}{2\sqrt{e^y}} \cdot e^y = \frac{e^y}{2\sqrt{e^y}} = \frac{1}{2}\sqrt{e^y}$$

$$Z_y = 2y\cdot\sqrt{e^y} + (x+y^2)\cdot\frac{1}{2\sqrt{e^y}}\cdot e^y = 2y\sqrt{e^y} + \frac{1}{2}(x+y^2)\sqrt{e^y} =$$

$$\frac{e^x}{(e^x)^{\frac{1}{2}}}$$

$$= \sqrt{e^y}\left(2y + \frac{x}{2} + \frac{y^2}{2}\right)$$

$$Z_{yy} = \frac{1}{2\sqrt{e^y}}\cdot e^y\left(2y+\frac{x}{2}+\frac{y^2}{2}\right) + \sqrt{e^y}\left(2+\frac{2y}{2}\right) =$$

$$= \frac{1}{2}\sqrt{e^y}\left(2y+\frac{x}{2}+\frac{y^2}{2}\right) + \sqrt{e^y}(y+2) =$$

$$= \sqrt{e^y}\left(y+\frac{x}{4}+\frac{y^2}{4}+y+2\right) = \sqrt{e^y}\left(2y+2+\frac{x}{4}+\frac{y^2}{4}\right)$$

$$\boxed{d^2z = \sqrt{e^y}dxdy + \sqrt{e^y}\left(2y+2+\frac{x}{4}+\frac{y^2}{4}\right)d^2y}$$

(34) $z(x,y) = (x+y)\ln(1-x-y)$; $M(0,0)$

$$Z_x = \ln(1-x-y) + (x+y)\cdot\frac{1}{1-x-y}\cdot(-1) =$$

$$= \ln(1-x-y) - \frac{x+y}{1-x-y}$$

$$Z_{xx} = \frac{-1}{1-x-y} - \frac{(1-x-y)+(x+y)(-1)}{(1-x-y)^2} =$$

$$= \frac{-1+x+y-1+\cancel{x+y}-\cancel{x-y}}{(1-x-y)^2} = \frac{x+y-2}{(1-x-y)^2} = -2$$

$$Z_{xy} = -\frac{1}{1-x-y} - \frac{1-x-y+(x+y)(-1)}{(1-x-y)^2} =$$

$$= \frac{-1+x+y-1+\cancel{x+y}-\cancel{x-y}}{(1-x-y)^2} = \frac{x+y-2}{(1-x-y)^2} = -2$$

$$Z_y = \ln(1-x-y) - \frac{x+y}{1-x-y}$$

$$\boxed{d^2z = -2d^2x - 4dxdy - 2d^2y}$$

$$Z_{yy} = \frac{x+y-2}{(1-x-y)^2} = -2$$

(35.) $z(x,y) = x^4 + y^4 - (x-y)^2;\ M(1,-1)$

$z_x = 4x^3 - 2(x-y)$

$z_{xx} = 12x^2 - 2 = 10$

$z_{xy} = 2$

$z_y = 4y^3 + 2(x-y)$

$z_{yy} = 12y^2 - 2 = 10$

$$\boxed{d^2z = 10d^2x + 4dxdy + 10d^2y}$$

(36.) $z(x,y) = y\sin x + x\cos y;\ M(0,0)$

$z_x = y\cdot\cos x + \cos y$

$z_{xx} = -y\cdot\sin x = 0$

$z_{xy} = \cos x - \sin y = 1$

$z_y = \sin x - x\sin y$

$z_{yy} = -x\cos y = 0$

$$\boxed{d^2z = 2dxdy}$$

(37.) $\cancel{z}(x,y,z) = \dfrac{x^2y - yx^2}{x+y} = 0$?

(38.) $z(x,y,z) = \dfrac{x-y^2}{x^2+y^3}$

$$z_x = \frac{x^2+y^3-(x-y^2)\,2x}{(x^2+y^3)^2} = \frac{x^2+y^3-2x^2+2xy^2}{(x^2+y^3)^2} = \frac{-x^2+y^3+2xy^2}{(x^2+y^3)^2}$$

$$z_{xx} = \frac{(-2x+2y^2)(x^2+y^3)^2-(-x^2+y^3+2xy^2)\cdot 2(x^2+y^3)\cdot 2x}{(x^2+y^3)^4} =$$

$$= \frac{\cancel{(x^2+y^3)}\left((x^2+y^3)(2y^2-2x)-4x(-x^2+y^3+2xy^2)\right)}{(x^2+y^3)^{\cancel{4}\,3}} =$$

$$= \frac{2x^2y^2-2x^3+2y^5-2xy^3+4x^3-4xy^3-8x^2y^2}{(x^2+y^3)^3} =$$

$$= \frac{2x^3+2y^5-6x^2y^2-6xy^3}{(x^2+y^3)^3}$$

$$z_{xy} = \frac{(3y^2+4xy)(x^2+y^3)^2-(-x^2+y^3+2xy^2)\cdot 2(x^2+y^3)\cdot 3y^2}{(x^2+y^3)^4} =$$

$$= \frac{\cancel{(x^2+y^3)}\left((x^2+y^3)(3y^2+4xy)-6y^2(-x^2+y^3+2xy^2)\right)}{(x^2+y^3)^{\cancel{4}\,3}} =$$

$$= \frac{3x^2y^2 + 4x^3y + 3y^5 - 4xy^4 + 6x^2y^2 - 6y^5 + 12xy^4}{(x^2+y^3)^3}$$

$$= \frac{-3y^5 + 9x^2y^2 + 4x^3y + 8xy^4}{(x^2+y^3)^3}$$

$$z_y = \frac{-2y(x^2+y^3) - (x-y^2)\cdot 3y^2}{(x^2+y^3)^2} = \frac{-2x^2y - 2y^4 - 3xy^2 + 3y^4}{(x^2+y^3)^2} = \frac{y^4 - 3xy^2 - 2x^2y}{(x^2+y^3)^2}$$

$$z_{yy} = \frac{(4y^3 - 6xy - 2x^2)(x^2+y^3)^2 - (y^4 - 3xy^2 - 2x^2y)\cdot 2(x^2+y^3)\cdot 3y^2}{(x^2+y^3)^4}$$

$$= \frac{2\cancel{(x^2+y^3)}\left((x^2+y^3)(2y^3 - 3xy - x^2) - 3y^2(y^4 - 3xy^2 - 2x^2y)\right)}{(x^2+y^3)^{\cancel{4}\,3}} =$$

$$= \frac{2(2x^2y^3 - 3x^3y - x^4 + 2y^6 - 3xy^4 - x^2y^3 - 3y^6 + 9xy^4 + 6x^2y^3)}{(x^2+y^3)^3} =$$

$$= \frac{2(-x^4 - y^6 + 7x^2y^3 - 3x^3y + 6xy^4)}{(x^2+y^3)^3}$$

39. $u(x,y,z) = xyz$

$u_x = yz$ $\qquad u_{xy} = z$

$u_{xx} = 0$ $\qquad u_{xz} = y$

$u_y = xz$ $\qquad u_{yz} = x$

$u_{yy} = 0$ $\qquad \boxed{d^2z = 2z\,dx\,dy + 2y\,dx\,dz + 2x\,dy\,dz}$

$u_z = xy$

$u_{zz} = 0$

40. $u(x,y,z) = \frac{x}{y} + \frac{y}{z} + \frac{z}{x}$; $M(1,1,1)$

$u_x = \frac{1}{y} - \frac{z}{x^2}$

$u_{xx} = \frac{2z}{x^3} = 2$ $\qquad \boxed{d^2z = 2d^2x + 2d^2y + 2d^2z - 2dx\,dy - 2\,dx\,dz - 2\,dy\,dz}$

$u_{xy} = -\frac{1}{y^2} = -1$

$u_{xz} = -\frac{1}{x^2} = -1$

$u_y = -\frac{x}{y^2} + \frac{1}{z}$

$u_{yy} = +\frac{2x}{y^3} = +2$

$u_{yz} = -\frac{1}{z^2} = -1$

$u_z = -\frac{y}{z^2} + \frac{1}{x}$

$u_{zz} = +\frac{2y}{z^3} = -2$

41) $u(x,y,z)=xy+yz+zx$, $M(1,2,-1)$

$u_x=y+z$ $\quad u_y=x+z$ $\quad u_z=y+x$

$u_{xx}=0$ $\quad u_{yy}=0$ $\quad u_{zz}=0$

$u_{xy}=1$ $\quad u_{yz}=1$

$u_{xz}=1$

$$\boxed{d^2u=2dxdy+2dxdz+2dydz}$$

42) $u(x,y,z)=\sin x\sin y\sin z$

$u_x=\cos x\sin y\sin z$ $\quad u_y=\cos y\cdot\sin x\sin z$ $\quad u_z=\cos z\cdot\sin x\sin y$

$u_{xx}=-\sin x\sin y\sin z$ $\quad u_{yy}=-\sin y\cdot\sin x\cdot\sin z$ $\quad u_{zz}=-\sin x\sin y\sin z$

$u_{xy}=\cos x\cos y\sin z$ $\quad u_{yz}=\cos z\cdot\cos y\cdot\sin x$

$u_{xz}=\cos x\sin y\cos z$

$$\boxed{\begin{aligned}d^2u=&-\sin x\sin y\sin z(d^2x+d^2y+d^2z)+2\cos x\cos y\sin z\,dxdy+\\&+2\cos x\cos z\sin y\,dxdz+2\cos z\cos y\cdot\sin x\cdot dydz\end{aligned}}$$

43) $u(x,y,z)=\cos x\cos y\cos z$

$u_x=-\sin x\cdot\cos y\cdot\cos z$ $\quad u_y=-\cos x\sin y\cos z$ $\quad u_z=-\cos x\cos y\sin z$

$u_{xx}=-\cos x\cdot\cos y\cdot\cos z$ $\quad u_{yy}=-\cos x\cos y\cos z$ $\quad u_{zz}=-\cos x\cos y\cos z$

$u_{xy}=\sin x\sin y\cos z$ $\quad u_{yz}=\cos x\cdot\sin y\sin z$

$u_{xz}=\sin x\cos y\sin z$

$$\boxed{\begin{aligned}d^2u=&-\cos x\cdot\cos y\cos z(d^2x+d^2y+d^2z)+2\cdot\sin x\sin y\cos z\cdot dxdy+\\&+2\sin x\cos y\sin z\cdot dxdz+2\cdot\cos x\sin y\sin z\,dydz\end{aligned}}$$

44) $u(x,y,z)=\operatorname{ch}x\operatorname{ch}y\operatorname{ch}z$

$u_x=\operatorname{sh}x\operatorname{ch}y\operatorname{ch}z$ $\quad u_y=\operatorname{ch}x\operatorname{sh}y\operatorname{ch}z$ $\quad u_z=\operatorname{ch}x\operatorname{ch}y\operatorname{sh}z$

$u_{xx}=\operatorname{ch}x\operatorname{ch}y\operatorname{ch}z$ $\quad u_{yy}=\operatorname{ch}x\operatorname{ch}y\operatorname{ch}z$ $\quad u_{zz}=\operatorname{ch}x\operatorname{ch}y\operatorname{ch}z$

$u_{xy}=\operatorname{sh}x\operatorname{sh}y\operatorname{ch}z$ $\quad u_{yz}=\operatorname{ch}x\operatorname{sh}y\operatorname{sh}z$

$u_{xz}=\operatorname{sh}x\operatorname{ch}y\operatorname{sh}z$

$$\boxed{\begin{aligned}d^2u=&\operatorname{ch}x\cdot\operatorname{ch}y\cdot\operatorname{ch}z(d^2x+d^2y+d^2z)+2\operatorname{sh}x\operatorname{sh}y\operatorname{ch}z\cdot dxdy+\\&2\operatorname{sh}x\operatorname{ch}y\operatorname{sh}z\,dxdz+2\operatorname{ch}x\operatorname{sh}y\operatorname{sh}z\,dydz\end{aligned}}$$

(45) $u(x,y,z)=\sqrt{1-x^2-y^2-z^2}$

$u_x=\frac{-x}{\sqrt{1-x^2-y^2-z^2}}$

$u_{xx}=\frac{-\sqrt{1-x^2-y^2-z^2}+x\frac{1}{2\sqrt{1-x^2-y^2-z^2}}(-2x)}{1-x^2-y^2-z^2}=\frac{-2(1-x^2-y^2-z^2)+x}{2(1-x^2-y^2-z^2)^{3/2}}=$

$u_{xy}=\frac{x}{1-x^2-y^2-z^2}\cdot\frac{1}{2\sqrt{1-x^2-y^2-z^2}}\cdot(-2y)=\frac{-2xy}{2(1-x^2-y^2-z^2)^{3/2}}=\frac{-xy}{(1-x^2-y^2-z^2)^{3/2}}$

$u_{xz}=\frac{x}{1-x^2-y^2-z^2}\cdot\frac{1}{2\sqrt{1-x^2-y^2-z^2}}\cdot(-2z)=\frac{-xz}{(1-x^2-y^2-z^2)^{3/2}}$

$u_y=\frac{-y}{\sqrt{1-x^2-y^2-z^2}}$ $\qquad u_z=\frac{-z}{\sqrt{1-x^2-y^2-z^2}}$

$u_{yy}=\frac{-2(1-x^2-y^2-z^2)+y}{2(1-x^2-y^2-z^2)^{3/2}}$ $\qquad u_{zz}=\frac{-2(1-x^2-y^2-z^2)+z}{2(1-x^2-y^2-z^2)^{3/2}}$

$u_{yz}=\frac{-yz}{(1-x^2-y^2-z^2)^{3/2}}$

$$\boxed{d^2u=\frac{-2(1-x^2-y^2-z^2)+x}{2(1-x^2-y^2-z^2)^{3/2}}d^2x+\frac{2(1-x^2-y^2-z^2)+y}{2(1-x^2-y^2-z^2)^{3/2}}d^2y-\frac{2(1-x^2-y^2-z^2)-z}{2(1-x^2-y^2-z^2)^{3/2}}d^2z-\frac{2xy}{(1-x^2-y^2-z^2)^{3/2}}dxdy-\frac{2xz}{(1-x^2-y^2-z^2)^{3/2}}dxdz+\frac{2yz}{(1-x^2-y^2-z^2)^{3/2}}dydz}$$

(46) $u(x,y,z)=\frac{y}{x^2}+yz+z\sqrt{x}$; $M(1,1,2)$

$u_x=-\frac{2y}{x^3}+\frac{z}{2\sqrt{x}}$

$u_{xx}=\frac{6y}{x^4}+\frac{z}{2}\left(-\frac{1}{x}\cdot\frac{1}{2\sqrt{x}}\right)=\frac{6y}{x^4}-\frac{z}{4\sqrt{x^3}}=6-\frac{1}{2}=\frac{11}{2}$

$u_{xy}=-\frac{2}{x^3}=-2$

$u_{xz}=\frac{1}{2\sqrt{x}}=\frac{1}{2}$

$u_y=\frac{1}{x^2}+z$ $\qquad u_z=y+\sqrt{x}$

$u_{yy}=0$ $\qquad u_{zz}=0$

$u_{yz}=1$

$$\boxed{d^2u=\frac{11}{2}d^2x-4dxdy+dxdz+2dydz}$$

(47) $u(x,y,z) = \ln(x^2+y^2-z)$

$$u_x = \frac{2x}{x^2+y^2-z}$$

$$u_{xx} = \frac{2(x^2+y^2-z) - 2x \cdot 2x}{(x^2+y^2-z)^2} = \frac{2x^2+2y^2-2z-4x^2}{(x^2+y^2-z)^2} = \frac{-2x^2+2y^2-2z}{(x^2+y^2-z)^2}$$

$$u_{xy} = -\frac{4xy}{(x^2+y^2-z)^2}$$

$$u_{xz} = \frac{+2x}{(x^2+y^2-z)^2}$$

$$u_y = \frac{2y}{x^2+y^2-z}$$

$$u_{yy} = \frac{2x^2-2y^2-2z}{(x^2+y^2-z)^2}$$

$$u_z = -\frac{1}{x^2+y^2-z}$$

$$u_{zz} = -\frac{1}{(x^2+y^2-z)^2}$$

$$u_{yz} = \frac{2y}{(x^2+y^2-z)^2}$$

$$\boxed{d^2u = \frac{-2x^2+2y^2-2z}{(x^2+y^2-z)^2}d^2x + \frac{2x^2-2y^2-2z}{(x^2+y^2-z)^2}d^2y - \frac{1}{(x^2+y^2-z)^2}d^2z - \frac{8xy}{(x^2+y^2-z)^2}dxdy + \frac{4x}{(x^2+y^2-z)^2}dxdz + \frac{4y}{(x^2+y^2-z)^2}dydz}$$

(48) $u(x,y,z) = xy^2z^3$, $M(1,1,1)$

$u_x = y^2z^3$ $\quad$ $u_y = 2xyz^3$ $\quad$ $u_z = 3xy^2z^2$

$u_{xx} = 0$ $\quad$ $u_{yy} = 2xz^3 = 2$ $\quad$ $u_{zz} = 6xy^2z = 6$

$u_{xy} = 2yz^3 = 2$ $\quad$ $u_{yz} = 6xyz^2 = 6$

$u_{xz} = 3y^2z^2 = 3$

$$\boxed{d^2u = 2d^2y + 6d^2z + 4dxdy + 6dxdz + 12dydz}$$

(49) $u(x,y,z) = 2x^2+y^2-z^2-xz+5yz$

$u_x = 4x - z$ $\quad$ $u_y = 2y+5z$ $\quad$ $u_z = -2z-x+5y$

$u_{xx} = 4$ $\quad$ $u_{yy} = 2$ $\quad$ $u_{zz} = -2$

$u_{xy} = 0$ $\quad$ $u_{yz} = 5$

$u_{xz} = -1$

$$\boxed{d^2u = 4d^2x + 2d^2y - 2d^2z - 2dxdz + 10dydz}$$

(50) $u(x,y,z) = x^3 - \frac{x}{2y} + \frac{e^x}{x+z}$; $M(0,1,1)$

$u_x = 3x^2 - \frac{1}{2y} + \frac{e^x(x+z) - e^x}{(x+z)^2} = 3x^2 - \frac{1}{2y} + \frac{e^x(x+z-1)}{(x+z)^2}$

$u_{xx} = 6x + \frac{(e^x(x+z-1)+e^x)(x+z)^2 - e^x(x+z-1)\cdot 2(x+z)}{(x+z)^4} =$

$= 6x + \frac{e^x\cancel{(x+z)}\left((x+z)(x+z-1)+x+z-2(x+z-1)\right)}{(x+z)^{\cancel{4}\,3}} =$

$= 6x + \frac{e^x(x^2+xz-x+xz+z^2-z+x+z-2x-2z+2)}{(x+z)^3} =$

$= \cancel{6x}^{0} + \frac{e^x(\cancel{x^2}+z^2+2\cancel{xz}-\cancel{2x}-2z+2)}{(x+z)^3} = 1$

$u_{xy} = \frac{1}{2y^2} = \frac{1}{2}$

$u_{xz} = \frac{e^x(x+z)^2 - e^x(x+z-1)\cdot 2(x+z)}{(x+z)^4} = \frac{e^x\cancel{(x+z)}\left(x+z-2(x+z-1)\right)}{(x+z)^{\cancel{4}\,3}} =$

$= \frac{e^x(x+z-2x-2z+2)}{(x+z)^3} = \frac{e^x(2-x-z)}{(x+z)^3} = 1$

$-\frac{e^x}{(x+z)^2}$ $\frac{2e^x}{(x+z)^3}$ $2(x+z)$

$u_y = \frac{x}{2y^2}$

$u_{yy} = -\frac{2x}{2y^3} = -\frac{x}{y^3} = 0$

$\frac{e^x\cdot 2(x+z)}{(x+z)^4}$ $\frac{2e^x}{x+}$

$u_{yz} = 0$

$u_z = -\frac{e^x}{(x+z)^2}$ $\frac{1}{x^2} = -\frac{2}{x^3}$ $-e^x\left(\frac{1}{(x+z)^2}\right)$

$u_{zz} = \frac{2e^x}{(x+z)^{3}} \cdot \cancel{2(x+z)} = \frac{2e^x}{(x+z)^3} = \cancel{\frac{1}{4}}\ 2$ $-e^x\cdot\left(-\frac{2}{(x+z)^3}\right)(2(x+z))$

$\boxed{d^2u = d^2x + \cancel{h}\,d^2z + dxdy + 2dxdz}$

$\frac{4e^x(x+z)}{(x+z)^4}$ $\frac{2e^x}{(x+z)^2}$

(51) $u(x,y,z) = \ln(6-2x^2-3y^2-6z^2)$

$\frac{-e^x}{(x+z)^2}$ $= \frac{+e^x\cdot 2(x+z)}{(x+z)^4} = \frac{2e^x}{(x+z)^3}$

$u_x = \frac{-4x}{6-2x^2-3y^2-6z^2}$

$u_{xx} = \frac{-4(6-2x^2-3y^2-6z^2)+4x(-4x)}{(6-2x^2-3y^2-6z^2)^2} = \frac{-24+8x^2+12y^2+24z^2-16x^2}{(6-2x^2-3y^2-6z^2)^2} = \frac{-24-8x^2+12y^2+24z^2}{(6-2x^2-3y^2-6z^2)^2}$

$u_{xy} = \frac{4x}{(6-2x^2-3y^2-6z^2)^2}\cdot(-6y) = \frac{-24xy}{(6-2x^2-3y^2-6z^2)^2}$

$u_{xz} = \frac{-48zx}{(6-2x^2-3y^2-6z^2)^2}$

$u_{yy} = \frac{-6(6-2x^2-3y^2-6z^2)+6y(-6y)}{(6-2x^2-3y^2-6z^2)^2} =$

$u_y = \frac{-6y}{6-2x^2-3y^2-6z^2}$

$= \frac{-36+12x^2+18y^2+36z^2-36y^2}{(6-2x^2-3y^2-6z^2)^2} =$

$u_{yz} = \frac{-72yz}{(6-2x^2-3y^2-6z^2)^2}$ $= \frac{-36+12x^2-18y^2+36z^2}{(6-2x^2-3y^2-6z^2)^2}$

$$u_z = \frac{-12z}{6-2x^2-3y^2-6z^2}$$

$$u_{zz} = \frac{-12(6-2x^2-3y^2-6z^2)-12z\cdot 12z}{(6-2x^2-3y^2-6z^2)^2} =$$

$$= \frac{-72+24x^2+36y^2+72z^2-144z^2}{(6-2x^2-3y^2-6z^2)^2} =$$

$$= \frac{-72+24x^2+36y^2-72z^2}{(6-2x^2-3y^2-6z^2)^2} = \frac{-12(6-2x^2-3y^2+6z^2)}{(6-2x^2-3y^2-6z^2)^2}$$

$$\boxed{\begin{aligned} d^2u = &\frac{24-8x^2+12y^2+24z^2}{(6-2x^2-3y^2-6z^2)^2}d^2x + \frac{-36+12x^2-18y^2+36z^2}{(6-2x^2-3y^2-6z^2)^2}d^2y + \\ &+ \frac{12(6-2x^2-3y^2+6z^2)}{(6-2x^2-3y^2-6z^2)^2}d^2z - 2\frac{24xy}{(6-2x^2-3y^2-6z^2)^2}dxdy - \\ &- 2\frac{48xz}{(6-2x^2-3y^2-6z^2)^2}dxdz - 2\frac{72yz}{(6-2x^2-3y^2-6z^2)^2}dydz \end{aligned}}$$

(52) $u(x,y,z) = \sqrt{x^2+y^2-z^2}$; $M(1,1,1)$

$$u_x = \frac{x}{\sqrt{x^2+y^2-z^2}}$$

$$u_{xx} = \frac{\sqrt{x^2+y^2-z^2} - x\frac{1}{2\sqrt{x^2+y^2-z^2}}\cdot 2x}{x^2+y^2-z^2} = \frac{2x^2+2y^2-2z^2-x}{2(x^2+y^2-z^2)^{3/2}} = \frac{1}{2}$$

$$u_{xy} = -\frac{x}{x^2+y^2-z^2}\cdot\frac{1}{2\sqrt{x^2+y^2-z^2}}\cdot 2y = -\frac{xy}{(x^2+y^2-z^2)^{3/2}} = -1$$

$$u_{xz} = \frac{xz}{(x^2+y^2-z^2)^{3/2}} = 1$$

$$u_y = \frac{y}{\sqrt{x^2+y^2-z^2}}$$

$$u_{yy} = \frac{2x^2+2y^2-2z^2-y}{2(x^2+y^2-z^2)^{3/2}} = \frac{1}{2}$$

$$u_{yz} = -\frac{yz}{(x^2+y^2-z^2)^{3/2}} = -1$$

$$u_z = -\frac{z}{\sqrt{x^2+y^2-z^2}}$$

$$u_{zz} = \frac{2x^2+2y^2-2z^2+z}{2(x^2+y^2-z^2)^{3/2}} = \frac{3}{2}$$

$$\boxed{d^2u = \frac{1}{2}d^2x + \frac{1}{2}d^2y + \frac{3}{2}d^2z - 2dxdy + 2dxdz - 2dydz}$$

(53) $u(x,y,z) = x^2+y^2+z^2-2xy+xz+y$

$u_x = 2x-2y+z$ $u_y = 2y-2x+1$ $u_z = 2z+x$

$u_{xx} = 2$ $u_{yy} = 2$ $u_{zz} = 2$

$u_{xy} = -2$ $u_{yz} = 0$

$u_{xz} = 1$

$$\boxed{d^2u = 2d^2x+2d^2y+2d^2z-4dxdy+2dxdz}$$

(54) $u(x,y,z) = \frac{x}{y+z}+\frac{y}{z+x}+\frac{z}{x+y}$

$u_x = \frac{1}{y+z}-\frac{y}{(z+x)^2}-\frac{z}{(x+y)^2}$

$u_{xx} = \frac{2y}{(z+x)^3}+\frac{2z}{(x+y)^3}$

$u_{xy} = -\frac{1}{(y+z)^2}-\frac{1}{(z+x)^2}+\frac{2z}{(x+y)^3}$

$u_{xz} = -\frac{1}{(y+z)^2}+\frac{2y}{(z+x)^3}-\frac{1}{(x+y)^2}$

$u_y = -\frac{x}{(y+z)^2}+\frac{1}{z+x}-\frac{z}{(x+y)^2}$

$u_{yy} = \frac{2x}{(y+z)^3}+\frac{2z}{(x+y)^3}$

$u_{yz} = \frac{2x}{(y+z)^3}-\frac{1}{(z+x)^2}-\frac{1}{(x+y)^2}$

$u_z = -\frac{x}{(y+z)^2}-\frac{y}{(z+x)^2}+\frac{1}{x+y}$

$u_{zz} = \frac{2x}{(y+z)^3}+\frac{2y}{(z+x)^3}$

$\Rightarrow$ ~~X~~, ~~26~~ ?

(1.) $z(x,y) = x\sqrt{x+y^2}$, $M(3,1)$

$$z_x = \sqrt{x+y^2} + x\cdot\frac{1}{2\sqrt{x+y^2}} = \frac{2x+2y^2+x}{2(x+y^2)^{\frac12}} = \frac{3x+2y^2}{2(x+y^2)^{\frac12}}$$

$$z_{xx} = \frac{3\cdot 2(x+y^2)^{\frac12} - (3x+2y^2)\cdot\cancel{2}\,\frac12(x+y^2)^{-\frac12}}{4(x+y^2)} =$$

$$= \frac{6\sqrt{x+y^2} - (3x+2y^2)\cdot\cancel{2}\,\frac{1}{\cancel{2}\sqrt{x+y^2}}}{4(x+y^2)} =$$

$$= \frac{6x+6y^2-3x-2y^2}{4(x+y^2)^{3/2}} = \frac{3x+4y^2}{4(x+y^2)^{3/2}} = \frac{9+4}{4\cdot 8} = \frac{13}{32}$$

$z = f(u,v)$

$u = u(x,y)$

$v = v(x,y)$

$z_x = f_u u_x + f_v v_x$

$z_y = f_u u_y + f_v v_y$

(1.) $g(x,y) = f(2x^3+3y^2)$ $\qquad u = 2x^3+3y^2$

$y\cdot\frac{\partial g}{\partial x} = x^2\frac{\partial g}{\partial y}$ $\qquad u_x = 6x^2$

$\qquad u_y = 6y$

$y\cdot g_x = x^2\cdot g_y$

$g_x = \frac{\partial g}{\partial x} = \frac{\partial f}{\partial u}\cdot\frac{\partial u}{\partial x} = f_u\cdot u_x = f_u\cdot 6x^2$

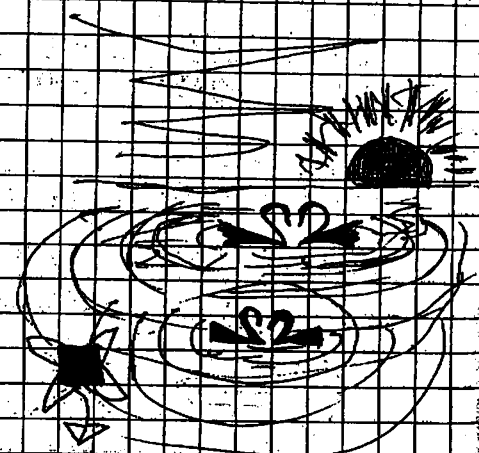
$g_x = f_u\cdot 6x^2$

$g_y = \frac{\partial g}{\partial y} = \frac{\partial f}{\partial u}\cdot\frac{\partial u}{\partial y} = f_u\cdot u_y = f_u\cdot 6y$

$g_y = f_u\cdot 6y$

$y\cdot f_u\cdot 6x^2 = x^2\cdot f_u\cdot 6y$

$f_u\cdot y\cdot 6x^2 = f_u\cdot y\cdot 6x^2$ ✓



(2.)

$z = f(u,v)$

$u = x+y$ $\qquad \frac{\partial^2 z}{\partial x\partial y} = \frac{\partial^2 z}{\partial u^2} - \frac{\partial^2 z}{\partial v^2}$ $\qquad z_{xy} = z_{uu} - z_{vv}$

$v = x-y$

$$\frac{\partial^2 z}{\partial x\partial y} = \cancel{\frac{f_u}{u_x}}\,u_x = \frac{\partial f}{\partial u}\cdot\frac{\partial u}{\partial x}$$

(25.) 1.2

$z(x,y) = x\,sh y + y\,ch x$; $M(0,0)$

$z_x = sh y + y\,sh x = \frac{e^y - e^{-y}}{2} + y \cdot \frac{e^x - e^{-x}}{2} = 0$

$z_y = x\,ch y + ch x = x \cdot \frac{e^y + e^{-y}}{2} + \frac{e^x + e^{-x}}{2} = x+1$

$\boxed{dz = (x+1)\,dy} \Rightarrow \boxed{dz = dy}$

$$\boxed{\begin{aligned} sh x &= \frac{e^x - e^{-x}}{2} \\ ch x &= \frac{e^x + e^{-x}}{2} \end{aligned}}$$

(25.) 1.3

$z(x,y) = x\,sh y + y\,ch x$; $M(0,0)$

$z_x = sh y + y\,sh x$

$z_{xx} = y\,ch x = y \cdot \frac{e^x + e^{-x}}{2} = y = 0$

$z_{xy} = ch y + sh x = \frac{e^y + e^{-y}}{2} + \frac{e^x - e^{-x}}{2} = 1$

$z_y = x\,ch y + ch x$

$z_{yy} = x\,sh y = x \cdot \frac{e^y - e^{-y}}{2} = 0$

$\boxed{d^2 z = 2\,dx\,dy}$

(26.) 1.3

$z(x,y) = x\,ch(2y-x)$; $M(0,0)$

$z_x = ch(2y-x) + x\,sh(2y-x)(-1) = ch(2y-x) - x\,sh(2y-x)$

$z_{xx} = -sh(2y-x) - sh(2y-x) + x\,ch(2y-x) =$

$= x\,ch(2y-x) - 2\,sh(2y-x) = 0$

$z_{xy} = sh(2y-x) \cdot 2 - x \cdot ch(2y-x) \cdot 2 = 0$

$z_y = 2x\,sh(2y-x)$

$z_{yy} = 4x\,ch(2y-x) = 0$

$\boxed{d^2 z = 0}$